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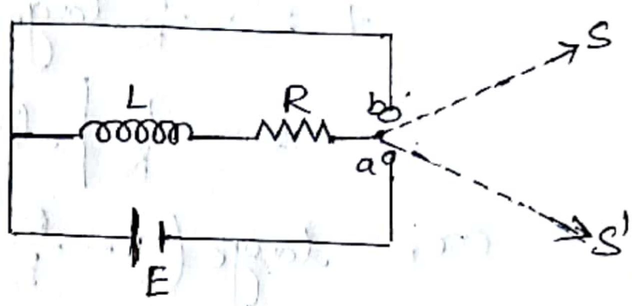
# Growth of Current in L-R Circuit.

Let us consider a circuit containing an inductance L and a wire resistance R in series when a cell of constant e.m.f. E is connected in series with it. When the circuit is closed by throwing the switch S to a point 'a' a self induction e.m.f is set up in the coil which opposes the applied e.m.f. Hence the current does not reach its final steady value E/R instantaneously. The current grows at a rate depending upon the inductance and resistance of circuit.




Let I be the current at any instant t.

$\therefore$ Rate of growth of current $= \frac{dI}{dt}$

$\therefore$ Induced e.m.f. across the coil $= -L\frac{dI}{dt}$

$\therefore$ Effective e.m.f. $= E - L\frac{dI}{dt}$

But the effective e.m.f = potential sets across the resistance $R = IR$

$$\therefore E - L\frac{dI}{dt} = RI$$

$$\therefore E - RI = L\frac{dI}{dt}$$

$$\therefore dt = \frac{L\,dI}{E - RI}$$

$$\int dt = \frac{L}{E-RI}\int dI$$

or, $t = -\frac{L}{R}\log_e(E-RI) + C$ ——— (1)

where $C$ is a constant of integration At $t=0, I=0$

$\therefore C = \frac{L}{R}\log_e(E)$

$\therefore$ from equation (1) we have

$$t = -\frac{L}{R}\log_e(E-RI) + \frac{L}{R}\log_e(E)$$

$$= -\frac{L}{R}\left[\log_e\left(\frac{E-RI}{E}\right)\right]$$

or, $\log_e\left(1-\frac{R}{E}\cdot I\right) = -\frac{R}{L}\cdot t$

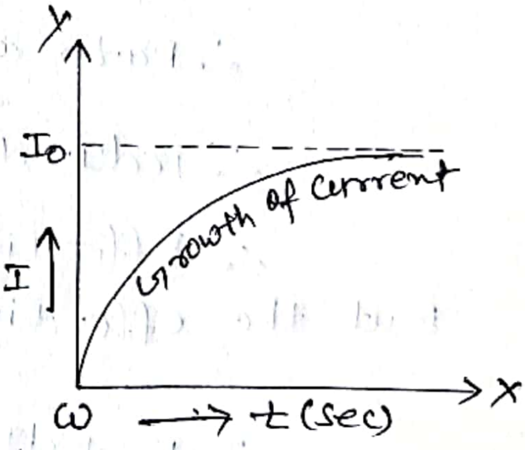
or, $1-\frac{RI}{E} = e^{-\frac{R}{L}\cdot t}$

or, $\frac{RI}{E} = 1-e^{-(R/L)t}$

$\therefore I = \frac{E}{R}\left(1-e^{-\left(\frac{R}{L}\right)t}\right)$

$\therefore I = I_0\left(1-e^{-(R/L)\cdot t}\right)$ ——— (2)

This equation shows that the current in the circuit rises according to an exponential law as shown in figure.




Differentiating eqn (2)

$$\frac{dI}{dt} = -I_0\left(-\frac{R}{L}\right)e^{-(R/L)t}$$

$$= I_0\left(\frac{R}{R}\right)e^{-(R/L)t}$$

From equation (2)

$$\frac{I}{I_0} = 1-e^{-(R/L)t}$$

or, $e^{-(R/L)t} = \left(\frac{I_0-I}{I_0}\right)$, $\therefore \frac{dI}{dt} = I_0\left(\frac{I-I_0}{I_0}\right)\frac{R}{L} = \frac{R}{L}(I-I_0)$ ——— (3)

This is clear that the rate of growth of current depends only the value of $R/L$. The quantity $L/R$ is called the time constant of the circuit and is denoted by $\lambda$.

$\therefore \lambda = \frac{L}{R}$

from eqn (2) we have, $I = I_0(1-e^{-t/\lambda})$ ——— (4)